Linear Programming

1. Define 3 variables.
2. Write a statement that tells what is to be maximized or minimized.
3. List the constraints (restrictions).
4. Label the axes.
    a. Graph the constraints (mostly using intercepts).
    b. Shade intersected areas.
    c. Label points of intersection as you graph.
5. Write the particular equation that the problem asks you to solve.
    a. Graph this line. (Sliding line)
    b. Slide this line to find the maximum or minimum point.

1. Jake Garcia opens a music shop in which he sells guitars and basses. He wants to find out the maximum amount of money he may have to borrow to purchase the instruments.

    A. Each bass will cost him $900 and each guitar will cost him $750. He buys B basses and G guitars. Write an equation for the total number of dollars, C, he will have to spend.

    B. Jake has certain restrictions on the numbers of each kind of instrument he can stock.
        1. His store is small, so he can buy no more than 50 instruments, total.
        2. Because guitars are more popular than basses, the number of guitars must be at least twice the number of basses.
        3. To get started, he must buy at least 17 guitars and at least 5 basses.
        4. Write a system of inequalities expressing these restrictions.

    C. Plot the graph of the system in part (B). Lightly shade the intersected areas.

    D. Write an inequality stating that Jake spends at least $36,000 on instruments. (sliding line) Darken the part of the feasible region in part (C) for which Jake spends at least $36,000.

    E. Slide the line in (D) in the greater than direction (keep the line parallel to the original line) until you reach the very last point of intersection plotted. This will be the maximum point. What number of basses and what number of guitars would give the maximum feasible cost? What would this maximum cost be?

2. Sabrina Burmeister is Chief Mathematician for Pedro Leum's Oil Refinery. Pedro can buy Texas oil, priced at $30 per barrel, and California oil, priced at $15 per barrel. He consults Sabrina to find out what is the most he might have to pay in a month for the oil the refinery uses.

   A. Define variables for the number of barrels of Texas oil and the number of barrels of California oil purchased in a month. Then write an equation expressing the total cost, c, of the oil in terms of these two variables.

   B. Sabrina finds the following restrictions on the amounts of oil that can be purchased in a month.
      1. The refinery can handle as much as 40,000 barrels per month.
      2. To stay in business, the refinery must process at least 18,000 barrels per month.
      3. California oil has 6 pounds of impurities per barrel. Texas oil has only 2 pounds of impurities per barrel. The most the refinery can handle is 120,000 pounds of impurities a month.
      4. Write a system of inequalities representing this information.

   C. Plot the graph of the system in part (B). Lightly shade the intersected area.

   D. Write an inequality saying Pedro spends at least $660,000 per month buying oil. (sliding line) Darken the part of the feasible region in part (C) which satisfies this inequality.

   E. Slide the line in (D) greater than direction (keep line parallel to the original line) until you reach the last point of intersection plotted. What is the maximum feasible amount Pedro might have to spend in a month? How much of each kind of oil would give this maximum amount?

3. Calvin Butterball is Chief Mathematician for Fly-By-Night Aircraft Corp. He is responsible for mathematical analysis of the manufacturing of the Company's two models of planes, the Sopwith Camel and the larger Sopwith Hippopotamus. Each department at Fly-By-Night has certain restrictions concerning the number of planes which can be manufactured per day.

a. Production: No more than 7 Hippopotami and no more than 11 Camels can be manufactured per day.
b. Shipping: No more than 12 planes, total, can be manufactured per day.
c. Sales: The number of Hippopotami manufactured per day must be no more than twice the number of Camels.
d. Labor: You must use more than 1000 man-hours of labor per day. (It takes 100 man-hours to manufacture each Camel and 200 man-hours to manufacture each Hippo.)

1. Select variables to represent the number of Camels and Hippopotami manufactured per day, write inequalities expressing each of the above restrictions, and draw a graph of the feasible region. Lightly shade the intersection area.

2. If Fly-By-Night makes a profit of$300 per Camel and $200 per Hippo, show the region on your graph in which the daily profit would be at least $3000.

3. How many Hippos and how many Camels should be produced per day to give the greatest feasible profit? What would this profit be?

4. Your mathematics club has arranged to earn some extra money by cleaning up Carr Park. The City Recreation Department agrees to pay each old member $10 and each new member $8 for the service. (The club did the same thing last year so the old members are experienced.)

   A. Define variables, then write an equation expressing the dollars the club earns in terms of the number of old and new members who work.

   B. The following facts restrict the number of students who can work:
      1. The number of old members is non-negative, and so is the number of new members.
      2. The club has at most 9 old members and at most 8 new members can work.
      3. The Department will hire at least 6 students, but not more than 15.
      4. There must be at least 3 new members.
      5. The number of new members must be at least 1/2 the number of old members, but less than 3 times the number of old members.
      6. Write inequalities for each of the above requirements.

   C. Draw a graph of the solution set of this system of inequalities. (Remember that the club members come only in integer quantities!)

   D. Based on your graph, is it feasible to do without any old members at all? EXPLAIN!

   E. Shade the portion of the feasible region in which the club would make at least $100.

   F. Draw a line on the graph showing the number of old and new members needed to make $160. Is it feasible to make $160? EXPLAIN!

   G. What number of old and new members would earn the maximum feasible amount? What would this amount be?

   H. What is the minimum feasible amount the club could earn?

   I. Suppose tradition was broken and the new members were paid more than the old members. What would be the maximum feasible earnings if new members get $12 and the old members get $10?

5. Suppose that you are Chief Mathematician for the Government's million dollar Vitamin Research Project. Your scientists have been studying the combined effects of Vitamin A and B on the human system, and have turned to you for analysis of their findings:

   a. The body can tolerate no more than 600 units per day of vitamin A, and no more than 500 units per day of vitamin B.
   b. The total number of units per day of the two vitamins must be between 400 and 1000, inclusive.
   c. Due to the combined effects of the two vitamins, the number of units per day of vitamin B must be more than 1/2 the number of units per day of vitamin A, but less than or equal to three times the number of units per day of vitamin A.

Let x and y be the number of units per day of vitamins A and B, respectively.
Answer the following questions:

1. Let c be the number of **<u>cents</u>** per day it costs you for vitamins. Write an equation expressing c in terms of x and y if vitamin A costs <u>0.06 cents</u> per unit (not $0.06!), and vitamin B costs <u>0.05 cents</u> per unit.

2. Write inequalities to represent each of the above requirements, and plot a graph of the solution set of the system.

3. What does a point lying in this solution set represent in the real world?

4. Plot lines on your graph showing where the cost per day is
   i. 60 cents
   ii. 30 cents
   iii. 15 cents

You might use different colored pencils to make these lines show up more distinctly.

5. Is it feasible to spend 60 cents per day oil vitamins? 30 cents per day? 15 cents per day? EXPLAIN WHY OR WHY NOT!!

6. What work describes the point at which the feasible cost is a minimum? Write the coordinates of this point, and find the minimum cost.

7. Craig Browning bakes cookies for the elementary school cookie sale. His chocolate chip cookies sell for $1.00 a dozen, and his oatmeal brownie cookies sell for $1.50 a dozen. He will bake up to 20 dozen chocolate chip cookies and up to 40 dozen oatmeal brownie cookies, but no more than 50 dozen cookies, total. Also, the number of oatmeal brownie cookies will be no more than 3 times the number of chocolate chip cookies. How many of each kind should Craig make in order for the elementary school to make the most money? How much money will this be?

   A. Define variables

   B. Write inequalities for each of the requirements

   C. Plot the graph of the feasible region

   D. Answer all questions

8. Monica Petry runs a gift shop in which she sells expensive trees at holiday time. Her supplier, Connie Furr, charges $80 for each real tree, and $160 for each artificial tree. She can buy between 20 and 90 real trees, inclusive, but up to 100 artificial trees. Connie can supply anywhere between 50 and 120 trees, total, but requires that the number of artificial trees ordered be at least half the number of real trees. Monica is hard-up for cash this year, and wants to invest the minimum feasible amount in trees. How many of each kind should she buy? What is her minimum feasible investment?

   A. Define variables

   B. Write inequalities for each of the requirements

   C. Plot the graph of the feasible region

   D. Answer all questions